# AN ANALYTICAL INVESTIGATION OF MECHANICAL VIBRATION FILTERS WITH NONLINEAR CHARACTERISTICS

BY

WILL J. WORLEY

B.S., University of Illinois, 1943
M.S., University of Illinois, 1945

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1952

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# AN ANALYTICAL INVESTIGATION OF MECHANICAL VIBRATION FILTERS WITH NONLINEAR CHARACTERISTICS

## Introduction

The problem of isolating a body on a suspension from the vibrations experienced by the base to which the suspension is attached is the subject of this paper. This problem is important in all tpyes of transportation vehicles, and in the shipping industry where the containers must isolate the merchandise from vibration and subsequent damage. These are but two of many similar applications where better vibration isolation characteristics are sought under the conditions where both steady state and transient vibrations must be considered.

The solution to this problem for a linear spring and linear damper is well known. The objective of this investigation is to determine whether a nonlinear spring and a nonlinear damper will improve the isolation characteristics of a suspension for steady state vibrations as well as for transient vibrations.

Previous investigators have studied various types of springs and dampers. A spring with a hyperbolic tangent load-deflection characteristic has been found by one investigator to have desirable vibration isolation features. Both this type of spring characteristic and the linear spring characteristic were used in this paper.

Dampers were selected with damping forces varying as a power of the absolute value of the velocity of the suspended body relative to the velocity of the base to which the suspension was attached. The exponent of the relative velocity was designated by n, thus viscous damping would be indicated by $n = 1$. Other values were $n = \frac{1}{2}$, 2, and 3. These values indicated damping forces which varied as the square root,

square, and cube of the absolute value of the velocity of the suspended body with respect to the velocity of the vibrating base. In addition to the above, two types of nonlinear unsymmetrical damping were studied. These types of damping were designated by $n = 2(-), \frac{1}{2}(+)$ and $n = \frac{1}{2}(-), 2(+)$. The first type indicates that the damping forces vary as the square of the relative velocity for negative values of the relative velocity and as the square root of the relative velocity for positive values. The second type is the reverse of the first type.

The equivalent viscous damping concept was used to evaluate the damping coefficient for the various nonlinear damping characteristics. The coefficients were selected to give equivalent damping at the natural frequency for the linear spring-mass system and for a small value of the excitation amplitude.

Damping indicated by $n = \frac{1}{2}$, 1, 2, 3, $(2(-), \frac{1}{2}(+))$, and $(\frac{1}{2}(-), 2(+))$ were investigated with the linear spring, with a hyperbolic tangent spring having symmetrical loading, and with a hyperbolic tangent spring having unsymmetrical loading. These conditions were studied for steady state as well as for transient vibrations.

The investigation was conducted at three different frequencies. The natural frequency of a spring-mass system having a linear spring, with a stiffness equal to initial stiffness of the hyperbolic tangent spring, was used as a reference frequency. Frequencies of one-half this value and twice this value were also used. In addition to varying the frequency, the amplitude of excitation of the base was varied for each condition at each frequency.

## Discussion Of Methods

The general equation which was solved was a second order nonlinear differential equation of the form $(d^2x/dt^2) + A \mid dx/dt \mid^n + B\,f(x) = Cf(t)$ where the A, B, and C are constants. The (dx/dt) term indicates the first

derivative of the displacement, x, with respect to time or the velocity. The $(d^2x/dt^2)$ term indicates the second derivative of x with respect to time or the acceleration.

The above equation was solved with the aid of a differential equation analyzer which is a mechanical-electrical machine suited to the solution of the above type of equation for both steady state and transient conditions. The machine plots the solution in terms of displacement-time and velocity-time diagrams.

Photographs of the original differential analyzer records were presented as well as tabulated values for maximum and minimum relative displacement and relative velocity for both the steady state and transient conditions. These photographs and tables were analyzed with the following results.

## Conclusions

The following conclusions were drawn for the steady state conditions.

1. The equivalent viscous damping concept is reliable for all types of damping considered in the investigation. The damping is equivalent to viscous damping at the resonant frequency when used with a linear spring.

2. The nonlinear symmetrically loaded spring was found to function in the linear range both below and above resonance. The amplitudes were sufficient at resonance to cause the spring to function in the nonlinear range. At resonance it was more effective in limiting the relative displacement amplitudes for all types of damping than the linear spring.

3. The nonlinear unsymmetrically loaded spring displays a desirable phase shift at resonance for all types of damping as compared with the linear spring. This phase shift results in part from the fact that the nonlinear spring is actually operating above its resonant frequency. The resonant frequency referred to throughout the paper applies to resonance for a linear spring-mass system.

4. Nonlinear unsymmetrical damping results in an unsymmetrical displacement-time wave form for both the linear and nonlinear springs. This asymmetry amounts to a shift in the mean position about which the body oscillates and is most pronounced at twice the resonant frequency. This shift in the mean position varies in sense according to the amplitude and frequency of excitation. Asymmetry of this type does not necessarily result in unsymmetrical velocity-time curves under the same conidtions. Generally the resulting velocity-time curves are nearly symmetrical.

5. As the amplitude of excitation is increased, a phase shift results when damping designated by $n = \frac{1}{2}$ and $n = 2$ are used. The phase shift is opposite in sense for the two types of damping. Thus, when unsymmetrical damping, designated by $n = 2(-)$, $\frac{1}{2}(+)$ is used, the phase shifts oppose one another with the net effect that the phase shift is nearly zero.

The following conclusions were drawn for the transient conditions.

6. The nonlinear unsymmetrically loaded spring increases the period of the free oscillations under transient conditions and thus reduces the relative velocities and relative accelerations for frequencies equal to or greater than the resonant frequency for a linear spring-mass system. Below the resonant frequency the increased amplitudes, which the spring permits, more than offset the velocity reduction due to the increased period of free vibration.

7. Damping of the type designated by $n = \frac{1}{2}$ results in rapid decay of the free oscillations following a transient pulse. Damping designated by $n = 2$ or $n = 3$ results in a slow rate of decay. When two of these are combined to give $n = 2(-)$, $\frac{1}{2}(+)$ the rate of decay is relatively high.

For both steady state and transient vibrations the following conclusions were drawn.

8. All types of damping result in smaller relative displacement

amplitudes when used with the unsymmetrically loaded nonlinear spring at resonance or above resonance. Below resonance the linear spring resulted in smaller relative displacement amplitudes.

9. For all combinations of steady state and transient vibrations, damping designated by $n = 2(-), \frac{1}{2}(+)$ appeared to offer the best compromise. Above resonance, however, damping designated by $n = \frac{1}{2}(-), 2(+)$ was found to be more favorable in limiting steady state and transient relative displacement amplitudes.

These conclusions do not indicate a complete solution to the problem outlined in the introduction. They do, however, indicate very favorable trends which should stimulate further investigation of those types of nonlinear characteristics which showed the most favorable results.

## VITA

Will J. Worley was born on August 2, 1919, at Gibson City, Illinois. In September, 1938, he entered the College of Engineering at the University of Illinois, Urbana, Illinois, subsequently receiving the degree of Bachelor of Science, with honors, in Mechanical Engineering in June, 1943. He began graduate work at that time in the Department of Theoretical and Applied Mechanics in the same college, receiving the degree of Master of Science in June, 1945. From September, 1943, to September, 1949, he held an instructorship in Theoretical and Applied Mechanics. Since September, 1949, he has been an Assistant Professor of Theoretical and Applied Mechanics, at the University of Illinois, Urbana, Illinois.